The Sense part of Sensex

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***Index Terms*—Efficient Market Hypothesis, Efficiency of Sensex, Efficiency Tests for Financial Markets.**

***Abstract*—In this report, I examine the efficiency of the bellweather of Indian economy-Sensex. The sensex has been roaring and soaring for quite sometime now and there are doubts in the professional and academic circles that something may be wrong with the markets.**

I. Introduction

THE issue of efficient financial markets has been a matter of research and debate for a long time. Apart from the academics and the professionals, even the layman is interested in the efficiency of financial markets simply because the idea of hidden treasure lying undiscovered is an a enticing one. The issue is of more interest in the developing economies where it is a widely held belief that the participants are still learning the tricks of trade. A market is efficient if the asset prices are best indicator of all available information. The Efficient market hypothesis in effect says that the markets know it all. In its week form the hypothesis claims that all the knowledge from the history of prices is reflected and summarized in a single number - the current price of the asset. In its semi-strong form the hypothesis claims that the prices reflect not only the knowledge from historical prices but also from all publcily available sources.

Closely related to the idea of efficient markets is the Random Walk model. The underlying idea of this model is that the current prices contain all available information and the price change between two consecutive time periods is nothing but unpredictable error term. Since the model believes that all information has been 'digested' by the market the price changes between two consecutive time periods will come in only from new information that flows in during that time period. Since new information is random and unpredictable hence the difference in asset prices in these two time periods will also be random.The The Random Walk model posits that

$$P_t = P_{t-1} + \epsilon_{t-1} \tag{1}$$

with the assumption that

$$\epsilon \; i.i.d N(0,1) \tag{2}$$

where P_t and P_{t-1} are asset prices at time t and t-1 respectively and ϵ is the error term.

The autocorrelation function is

$$\rho(k) = \frac{Cov(r_t, r_{t+k})}{\sqrt{Var(r_t) * Var(r_{t+k})}} \tag{3}$$

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II. The Data

Daily data of Sensex from July 1,1997 to October 15, 2007 has been used in this study. This represents a good data set for a number of reasons. Ten years of data allows us to make significant conclusions about seasonality, day-of-week effect and cycles. As is evident from the graph below there was a recession in 2001-02 timeframe and the markets have been recovering since.

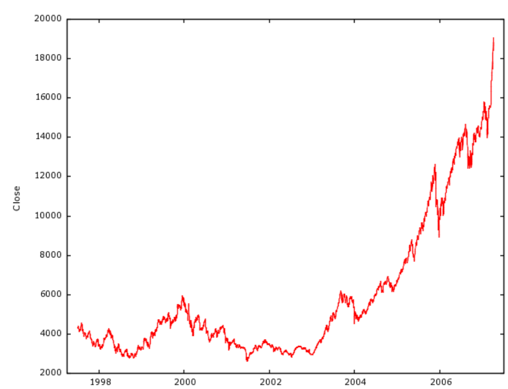

Figure 1. Time Series plot of the sensex

In our context it is more important to analyze the returns on the index because a price movement by itself is of little interest to an investor. For the purposes of this study daily returns (except for the variance ratio tests) have been used. The usual definition of returns suffers from the problem of assymetry. Hence, for our purposes the return is defined as

$$r(t) = ln[Index_t] - ln[Index_{t-1}] \tag{4}$$

We have purposely chosen this definition for the returns because the ordinary definition

$$r(t) = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{5}$$

suffers from problems of assymetry.

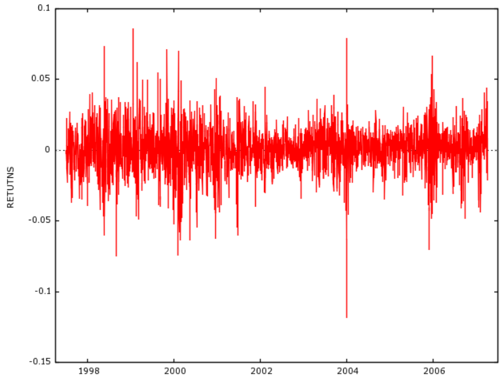

Figure 2. Time Series plot of returns

The figure above shows the daily returns on the index. A cursory look at the graph seems to convey that the returns are indeed random and distributed 'wildly' around 0. This would mean that returns are indeed random with an equlibrium value of 0, meaning on an average we are no better investing in the market than roaming wildly on the streets or better enjoying the beaches of Goa. Certainly, we can do a better analysis than this and find out how far is this proposition true.

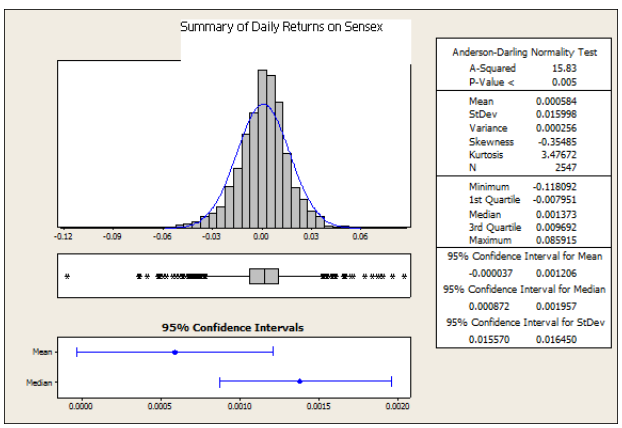

Figure 3. Summary plot of returns

The figure above shows that the mean of the returns is slightly better than zero - 0.000584. The 95% confidence interval for the mean is -0.000037 to 0.001206. Thank Heavens! Not all is lost. The returns are negatively skewed and have a kurtosis of 3.47. Traditionally, returns are expected to be leptokurtic, but for the sensex the returns seem to be mesokurtic form. It must be pointed here that these are terms are being loosely used here and there is absolutely no indications that the returns are normal. In any case, there is a mean reverting tendency. Is this against week form of efficiency? Mean reversion after all would imply that we can forecast the average returns on the sensex. The catch here is that the average is zero. The efficient market hypothesis is hence saying that on an average no one can earn super-normal profits. To this extent both ideas can be reconciled.

III. The Ljung Box Q Statistics

First let us examine the autocorrelations at various lags. The figure below indicates that there are significant autocorrelations at lag 1,6 and 9 at 95% confidence limit. This is a small indication that analyzing the performance of the sensex at these lags may lead to some future predictions of the market. There are significant partial autocorrelations at lags 1,4 and 9.

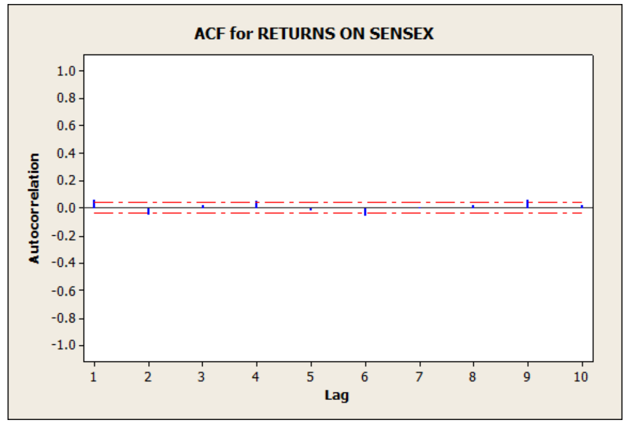

Figure 4. Autocorrelations for returns on Sensex

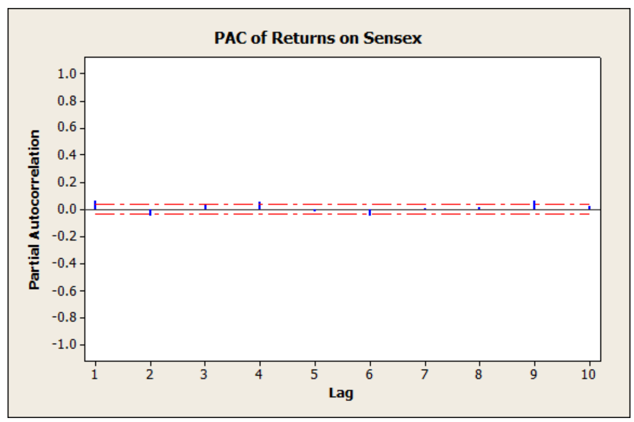

Figure 5. Partial Autocorrelations for returns on Sensex

By looking at the autocorrelations and partial autocorrelations, it can be seen that at least some of the correlations are significantly different from zero. This suggests that there might be some patterns that can be exploited by the investors. The null hypothesis for the test is that all autocorrelations are zero. The now obsolete Box-Pierce Q-Statistics, also called as Portmanateau Test is a test for white noise. The test was defined as

$$Q_{BP} = T \sum_{k=1}^{m} r_k^2 \qquad (6)$$

Where T is the number of observations and k is the number of lags under consideration. Unfortunately, this test did not perform well for small sample size. So for the purposes of this paper, I analyze the modified Q-Statistics, also called Ljung-Box Improvement. The modified Q-statistics is defined as

$$Q = T(T+2) \sum_{k=1}^{M} \frac{\rho(k)^2}{T-k} \qquad (7)$$

The null hypothesis can be written as

$$H_0 : \quad Q(l) = \chi^2(l) \tag{8}$$

This report examines the issues of efficiency and random walk of the Sensex.

Table I
ACF AND Q STATISTICS AT VARIOUS LAGS

Lag	ACF	t Statistics	Q Statistics
1	0.0643206	3.24612	10.5497
2	-0.0443749	-2.23029	15.5730
3	0.0233928	1.17344	16.9695
4	0.0589318	2.95457	25.8360
6	-0.0565556	-2.82527	34.4632
8	0.0202715	1.00950	35.5715
9	0.0610904	3.04101	45.1181
10	0.0254976	1.26466	46.7818

IV. DICKEY FULLER TEST

To examine the unit roots it is essential to take a stand on data generating process. In this report I take the stand that it is the returns which are inherently random. This would translater into testing unit roots for the natural logarithm of the asset prices. The Dickey Fuller Test is a test for $I(1)$. The test assumes an AR(1) process of the kind

$$P_t = \Upsilon P_{t-1} + \epsilon_t \tag{9}$$

and goes on to test if $\Upsilon = 1$. The other possible variants of the test are:

$$P_t = \zeta + \Upsilon P_{t-1} + \epsilon_t \tag{10}$$

$$P_t = \zeta + \Upsilon P_{t-1} + \eta t + \epsilon_t \tag{11}$$

and

$$P_t = \zeta + \Upsilon P_{t-1} + \eta t + \kappa t^2 + \epsilon_t \tag{12}$$

The equation is a random walk and adds an intercept to it. This means that this equation takes care of non-zero mean. The difference between and is that specifically takes a trend factor into the equation.

What is of special interest is the parameter Υ. $\Upsilon = 1$says that the series has unit root and is indeed a random walk. Therefore the test is

$$H_0 : \Upsilon \quad = \quad 1 \tag{13}$$
$$H_1 \, \Upsilon \quad \neq \quad 1 \tag{14}$$

Alternatively, defining $\Upsilon - 1$ as δ , the tests can be rewritten as

$$\triangle P_t = \delta P_{t-1} + \epsilon_t \tag{15}$$

$$\triangle P_t = \zeta + \delta P_{t-1} + \epsilon_t \tag{16}$$

$$\triangle P_t = \zeta + \delta P_{t-1} + \eta t + \epsilon_t \tag{17}$$

and

$$\triangle P_t = \zeta + \delta P_{t-1} + \eta t + \kappa t^2 + \epsilon_t \tag{18}$$

Now we test if $\delta = 0$. In exploratory data analysis and trend analysis, it was observed that most of the Indian Stock Market financial series have a quadratic trend. Hence, it seems just logical to test for $\delta = 0$ for a quadratic trend also.

We examine a number of aspects which are expected in an efficient market. These are

1) Examination of ACF and PACF characteristics
2) Unit Roots Test
3) Augmented Dickey Fuller Tests
4) Variance Ratio Tests (Lo and Mackinlay Tests)

The Dickey Fuller Test yielded the following result:

Table II
RESULTS OF DICKEY FULLER TEST CORRESPONDING TO VARIOUS EQUATIONS

Unit Root Test	δ	τ	p value
Without Constant	6.61388e-005	1.7861	0.9826
With Constant	0.000724548	1.12798	0.9978
With constant and trend	-0.00113571	-1.09654	0.9283
With constant and quadratic trend	-0.00424289	-2.3921	0.6292

This indicates that $\delta \neq 0$ because the p value is significant and the coefficient estimate is not 0. It must be noted that we don't expect a perfect value of 0, but how far is -0.004 from 0 is still an open question.

V. AUGMENTEDT DICKEY FULLER TEST

The ADF simply goes p steps further and considers a AR(p) process

$$\triangle P_t = \delta P_{t-1} + \sum_{i=1}^{p} \beta_i \triangle P_{t-i} + \epsilon_t \tag{19}$$

$$\triangle P_t = \zeta + \delta P_{t-1} + \sum_{i=1}^{p} \beta_i \triangle P_{t-i} + \epsilon_t \tag{20}$$

$$\triangle P_t = \zeta + \delta P_{t-1} + \sum_{i=1}^{p} \beta_i \triangle P_{t-i} + \eta t + \epsilon_t \tag{21}$$

$$\triangle P_t = \zeta + \delta P_{t-1} + \sum_{i=1}^{p} \beta_i \triangle P_{t-i} + \eta t + \kappa t^2 + \epsilon_t \tag{22}$$

Here we consider Augmented Dickey Fuller test at lag 9 (Criteria used: minimization of AIC)

Table III
RESULTS OF AUGMENTED DICKEY FULLER TEST AT LAG 9 CORRESPONDING TO VARIOUS EQUATIONS

Unit Root Test	δ	τ	p value
Without Constant	6.37827e-005	1.71875	0.9798
With Constant	0.000642501	0.992912	0.9966
With constant and trend	-0.00116201	-1.11785	0.9247
With constant and quadratic trend	-0.00445235	-2.49939	0.569

VI. VARIANCE RATIO TEST

The variance ratio tests developed by Lo & McKinlay(1989) are robust and powerful tests to detect the efficiency of a market. The test is described briefly here. The figure VI gives a general sense of relationship between returns and the lag. As can be seen in the plot below the returns 'flatten out' as the return period under consideration increases.

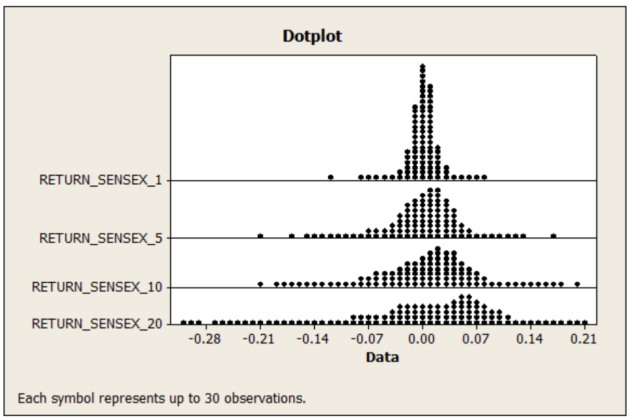

Figure 6. Comparison of Returns at 1 day, 5 days, 10 days and 20 days

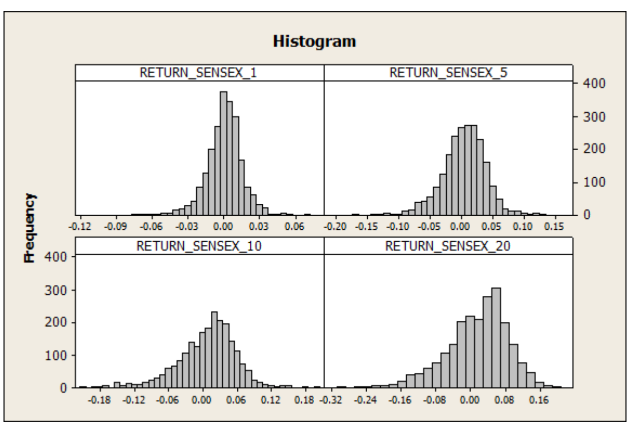

Figure 7. Distribution of returns at 1 day, 5 days, 10 days and 20 days

A. Lo & Mackinlay Test

The test is based on the premise that if a time series is indeed random walk, the variance of $P_t - P_{t-1}$ should be $\frac{1}{\mathbb{N}}$ times the variance of $P_t - P_{t-\mathbb{N}}$. Hence, we have the variance ratio defined as:

$$VR(\aleph) = \frac{Var[r_t(\aleph)]}{\aleph * Var[r_t(1)]} \quad (23)$$

Formally, the null hypothesis can be stated as

$$H_{0:}\ VR(\aleph) = \frac{Var[r_t(\aleph)]}{\aleph * Var[r_t(1)]} = 1 \quad (24)$$

Let

$$\widehat{\mu} = \frac{P_\aleph - P_0}{\aleph} \quad (25)$$

The unbiased estimator of variance at lag 1 is then defined as:

$$\bar{\sigma_1^2} = \frac{1}{\aleph}\sum_{k=1}^{k=\aleph}(P_k - P_{k-1} - \widehat{\mu})^2 \quad (26)$$

Similarly, at lag l the variance is defined as

$$\bar{\sigma_l^2} = \frac{1}{l(\aleph - \frac{\aleph}{l} + 1)(1 - \frac{1}{\aleph})}\sum_{k=l}^{k=\aleph}(P_k - P_{k-l} - l\widehat{\mu})^2 \quad (27)$$

The null hypothesis can now be restated as

$$H_{0:}\ VR(l) = \frac{\bar{\sigma_l^2}}{\bar{\sigma_1^2}} = 1 \quad (28)$$

Length	Variance	Variance Ratio
1	0.000248365	
2	0.000261738	1.053845211
4	0.000261091	1.051238448
8	0.000264619	1.065445441
16	0.000274351	1.104630323

Table IV
VARIANCE RATIOS FOR SENSEX AT VARIOUS LAGS

Variance Ratios.bmp

Figure 8. The movement of Variance Ratio with the number of lags

VII. HURST EXPONENT - LONH TERM MEMORY

The Hurst Exponent which measures the long term memory of the market comes out to be 0.560028 at lag 9. This immediately has two strong conclusions. First of all the returns series is not a random walk and second the series is not anti-persistent meaning returns are not mean-reverting!

VIII. CONCLUSION

IX. SUMMARY

The futures market are not efficient!

1) PHILLIPS PERRON TEST
2) KPSS STATIONARITY TEST
3) HURST EXPONENT - LONG TERM MEMORY OF THE MARKET

REFERENCES

[1]
[2]

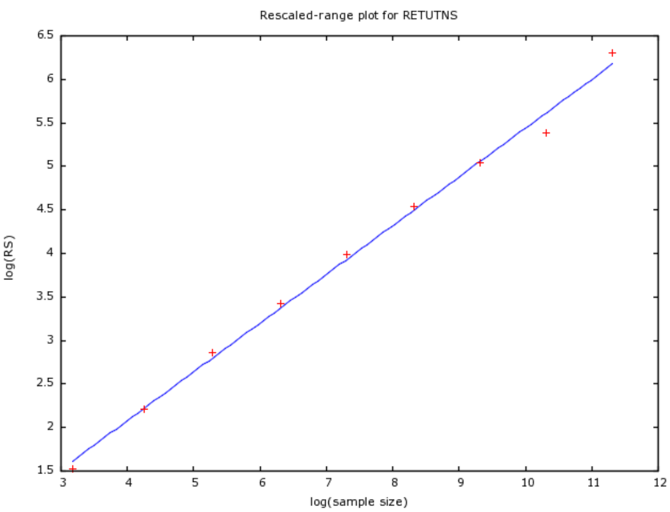

Figure 9. Hurst Exponent for Returns Series